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INTENSITY INTERFEROMETRY IN THE SPATIAL DOMAIN (II)

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Ballistic Research Laboratories

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REPORT NO. 1668

SEPTEMBER 1973

INTENSITY INTERFEROMETRY IN THE SPATIAL DOMAIN (II)

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13. ABSTRACT

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Intensity interferometry, as developed by Hanbury Brown and Twiss for stellar observation, has shown relative insensitivity to atmospheric scintillation. However, with classical sources, the limitations placed on this technique by quantum noise and detector efficiency are severe. This situation is vastly improved when laser illumination is employed. Generalizing a form of the mutual coherence function, we derive the far-zone behavior of the mutual intensity function for an intermediate time average. This result is used to reconstruct the irradiance distribution of a spatially rough source. The far-field intensity distribution is recorded spatially for one time-resolution unit of the detector. The resulting spatial signal is autocorrelated and related to the intensity distribution over the source. Thus, without averaging in the time domain, a spatial Fourier-transform relation is derived between the far-field intensity correlation and the source irradiance, similar to the results of Hanbury Brown and Twiss.

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I. INTRODUCTION

In an earlier BRL Report under this title, we developed the mathematical relationship between the intensity distribution over a rough surface and the intensity correlations in the far field. Although intrinsic to the underlying mathematics, the assumption of gaussian statistics was not explicitly used. In an effort to extend the approach to sources of arbitrary degree of spatial coherence, methods of gaussian statistics have been applied to this study. This leads to a simpler mathematical development.

The final results of this report are nearly identical to the earlier work.* However, here the detailed development of the work of Marchand and Wolf is omitted. We have retained the original introduction so that this report may be self complete.

The history of intensity interferometry is rooted in the work of Hanbury Brown and Twiss. Their earliest investigations[1**] dealt with the problem of resolving stellar radio sources by a technique involving the correlation of the squared outputs of two receivers. The advantages are the reduction of certain kinds of experimental constraints as well as the comparative insensitivity of the method to atmospheric scintillation.[2] A preliminary conclusion reached at that time was that this technique of intensity correlation would not be applicable at optical frequencies because of limitations imposed by photon noise. However, in later work,[3] Hanbury Brown and Twiss showed that meaningful intensity correlations could be made at optical frequencies even with highly degenerate sources. The limitations placed on this approach by quantum noise and detector efficiency have been severe, calling for highly refined experimental technique.

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*BRL Report No. 1616

**References may be found on page ??.

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For laser illumination, the situation is very different. The signal-to-noise ratio can be typically increased by six orders of magnitude.[4] However, the statistics of the source must be considered in the measurement. The key to relating intensity correlations to some property involving field correlations lies in the assumption of gaussian statistics,[5] for which all higher moments are determined from the first and second. Single-mode lasers, though, are distinctly nongaussian in their statistics, and, therefore, cannot be described by theory framed for thermal sources. But with the addition of only a few axial modes, the field amplitude becomes nearly gaussian distributed.[6]

The principal formula used by Hanbury Brown and Twiss[7] to infer the diameter of a distant source shows that the time-averaged correlation of intensities at two points is equal to the product of a function involving the temporal characteristics of the source with the square of the spatial-Fourier transform of the source intensity distribution. If the intent of an intensity correlation experiment is to gain information concerning the source intensity distribution, then the source temporal statistics may be of little interest in themselves. Consideration of the temporal statistics is necessary if the intensity-product output of the detectors is averaged in the time domain (as it nearly always is) to overcome the limitations imposed by photon and detector noise and possibly reduce scintillation effects produced by transmission through the atmosphere.

Intensity interferometry can be understood as a two-point correlation of intensities following the squaring of the electric field at the detector. If the source is quasi-monochromatic, each differential element on the object emits a number of temporal modes that interfere with each other at the detector. If the detector has sufficient speed[8] to detect these beat frequencies, the amplitude and phase of the incoming intensity beats are utilized. It is commonly argued that in these circumstances, the random fluctuations of the temporal statistics from different points of the source cause the beat frequencies from

each source point to add incoherently at the detector. The time-averaged intensity correlation is then proportional to the squared spatial-Fourier transform of the source intensity. This approach gives essentially a squared version of the van Cittert-Zernike theorem.[9]

We suggest that the requirement of surface roughness at the source (to assure spatial incoherence) is sufficient to guarantee the incoherent addition of beat frequencies at the intensity detectors. Thus, if temporal noise (photon noise, time-dependent detector noise) is largely absent in a local spatial sense, as might be the case with a multi-axial-mode laser with photographic detection, then the intensity information might be gathered during one resolution time of the detector over a plane section normal to the direction of light propagation. Any noise arising in the process would be spatial, and might be averaged out by taking a sufficiently large area of spatial correlation. The reduction of atmospheric spatial noise would be similar to a process known as aperture averaging.[10] Film-grain noise would be extremely well averaged by the relatively large area of averaging.

The relative insensitivity of intensity interferometry to turbulence is due to the assumed dispersionless nature of the propagation medium. Because each temporal frequency sees the same refractive index, the differential (beat) frequencies remain unchanged. However, the spatial-Fourier-transform relation between the source and the far field is scaled as the average frequency, not the beat frequency, and thus the resolution afforded by optical frequencies is maintained.

The idea of examining spatial beat frequencies of second-order correlation is, of course, not new. Many classical field-correlation interferferometers, as well as holographic experiments are built on this principle, involving a spatial or time lag between interfering beams of the same source. More difficult is the spatial recording of beats from two independent sources, as demonstrated by Magyar and Mandel.[11]

We have examined the problem of relating far-field spatial intensity (fourth-order-field) correlations to the intensity distribution of the source, without here considering the limitations due to noise. Our approach has been completely classical, drawing on a straight-forward generalization of a method given recently by Marchand and Wolf.[12] Our notation is similar through the development of their Eq. (35).

II. THE INTERMEDIATE-AVERAGE MUTUAL COHERENCE FUNCTION

For a stationary scalar wave field, the mutual coherence function for the correlation of two space-time points is often written

$$\Gamma(\underline{x}_1,\underline{x}_2,\tau,T) \equiv \frac{1}{2T} \int_{-T}^{T} V_T(\underline{x}_1,t+\tau)\ V_T^*(\underline{x}_2,t)\ dt, \tag{1}$$

where $\underline{x}_n = x_n\hat{i} + y_n\hat{j}$, and the limits for the time integration are allowed to approach infinity. For this case, however, we wish to keep the parameter T finite, and by the subscripts indicate that we assume a knowledge of $V_T(\underline{x}_1,t+\tau)$ and $V_T{}^*(\underline{x}_2,t)$ only over the finite sample length 2T. We wish to call $\Gamma(\underline{x}_1,\underline{x}_2,\tau,T)$ the intermediate-average mutual coherence function and stress that, for arbitrary T or shift of origin, it may bear little resemblance to the mutual coherence function defined by the ensemble average.

Following Ref. 12, we represent $V(\underline{x},t)$ as the temporal Fourier transform of the complex analytic signal (where a factor of $(2\pi)^{-1}$ has been suppressed),

$$V_T(\underline{x},t) = \int_0^{\infty} v_T(\underline{x},\omega)\ \exp(-i\omega t)d\omega \quad \text{for } 0 \leqslant |t| \leqslant T \tag{2a}$$

$$= 0 \text{ otherwise} \tag{2b}$$

and

$$v_T(\underline{x},\omega) = \int_{-T}^{T} V_T(\underline{x},t)\ \exp(i\omega t)\ dt. \tag{2c}$$

Substituting Eq. (2a) into Eq. (1), interchanging the order of integration and time-averaging, and performing the time integration, we get

$$\Gamma(\underline{x}_1,\underline{x}_2,\tau,T) = \iint_0^\infty W_T(\underline{x}_1,\underline{x}_2,\omega_1,\omega_2)$$

$$\times \exp(-i\omega_1\tau)\ \mathrm{sinc}[(\omega_1-\omega_2)T]\ d\omega_1 d\omega_2, \tag{3}$$

where

$$\mathrm{sinc}\ x \equiv \frac{\sin x}{x} \tag{4}$$

and the function

$$W_T(\underline{x}_1,\underline{x}_2\ \omega_1,\omega_2) \equiv v_T(\underline{x}_1,\omega_1)\ v_T^*(\underline{x}_2\omega_2) \tag{5}$$

is termed the cross-spectral density. The subscript T here and later implies a function based on the electric-field statistics only for the particular sample 2T in length about the origin. The sinc function of Eq. (3) assumes the role of a low-pass filter. If T is very small, the two frequency variables of Eq. (3) are essentially independent and all cross terms are represented in the product of Eq. (5). These cross terms form a high-frequency spectral content. However, as T tends to infinity, the sinc function assumes the role of a δ function, constraining correlation to occur only between identical frequencies in the transform product and forcing the integral to a one-dimensional form. In the limit of large T, the filtered spectrum of Eq. (5) becomes the mean-square value (dc) of each temporal-frequency component in the signal.

Omitting here the details, the far zone form of the cross-spectral density function has been derived by the method of Marchand and Wolf with the exception that the parameter T is left finite. The resulting form of the intermediate-average mutual coherence function is

$$\Gamma(\underset{\sim}{x}_1\underset{\sim}{x}_2\tau,T)$$

$$= 4\pi^2 \cos\theta_1 \cos\theta_2 \iint_0^\infty \exp(-i\omega_1\tau)\ \mathrm{sinc}[(\omega_1-\omega_2)T]$$

$$\times\ k_1k_2 d\omega_1 d\omega_2\ \frac{\exp[i(k_1r_1-k_2r_2)]}{r_1r_2}$$

$$\times\ V_T\left(k_1 \frac{x_1}{r_1},\ k_1 \frac{y_1}{r_1};\ 0;\ k_2 \frac{x_2}{r_2},\ k_2 \frac{y_2}{r_2};\ 0;\ \omega_1,\omega_2\right) . \qquad (6)$$

where the spatial-frequency correlation function in the plane $z = 0$ is defined

$$\hat{v}_T\left(k_1 \frac{x_1}{r_1},\ k_1 \frac{y_1}{r_1};\ 0;\ \omega_1\right) \hat{v}_T{}^*\left(k_2 \frac{x_2}{r_2},\ k_2 \frac{y_2}{r_2};\ 0;\ \omega_2\right)$$

$$\equiv V_T\left(k_1 \frac{x_1}{r_1},\ k_1 \frac{y_1}{r_1};\ 0;\ k_2 \frac{x_2}{r_2};\ k_2 \frac{y_2}{r_2};\ 0;\ \omega_1,\omega_2\right) , \qquad (7)$$

and $\hat{v}_T(f,g;z;\omega)$ is related by a two-dimensional spatial-Fourier transform to the time-frequency representation of the electric field by

$$v_T(x,y,z;\omega) = \iint_0^\infty \hat{v}_T(f,g;z;\omega)\ \exp[i(fx+gy)]\ dfdg. \qquad (8)$$

Finally the circular functions give the direction cosines for the two correlated fields, while the arguments are the angles that the two directions make with the z axis; specifically

$$\frac{z_1}{r_1} \equiv \cos\theta_1 \quad \text{and} \quad \frac{z_2}{r_2} \equiv \cos\theta_2 . \qquad (9)$$

If T is allowed to approach infinity, Eq. (6) reduces to the form given in Ref. 12 using Eqs. (5) and (35). We are now ready to form the self-intensity function in the far field.

III. THE SELF-INTENSITY FUNCTION IN THE FAR FIELD

We now examine the form of the self-intensity in the far field by letting points $\underline{x}_1 = \underline{x}_2 = \underline{x}$ and then letting the time delay, τ, be zero. Under these conditions the mutual coherence function reduces to the self-intensity,[13] and using Eq. (6), we have

$$I(\underline{x},T) = 4\pi^2 \cos^2\theta \iint_0^\infty d\omega_1 d\omega_2 \frac{\omega_1\omega_2}{c^2}$$

$$\times \frac{\exp[i(k_1-k_2)r]}{r^2} \operatorname{sinc}[(\omega_1-\omega_2)T]$$

$$\times \hat{v}_T\left(k_1 \frac{x}{r}, k_1 \frac{y}{r}; 0; \omega_1\right) \hat{v}_T^*\left(k_2 \frac{x}{r}, k_2 \frac{y}{r}; 0; \omega_2\right), \tag{10}$$

where, as indicated earlier, the sinc function acts to suppress temporal frequencies in the cross spectrum higher than $\sim 1/(2T)$ Hz. We now utilize the linear transformation of the time-frequency variables (for which the jacobian is unity) defined by

$$\omega_1 - \omega_2 \equiv \rho \quad \text{and} \quad \frac{\omega_1+\omega_2}{2} \equiv \sigma . \tag{11}$$

Writing the ω variables in terms of these center-of-mass coordinates, we get

$$\omega_1 = \frac{2\sigma+\rho}{2} \quad \text{and} \quad \omega_2 = \frac{2\sigma-\rho}{2} , \tag{12}$$

which, when substituted into Eq. (10), gives

$$I(\underline{x},T) = \left(\frac{2\pi}{cr}\right)^2$$

$$\times \cos^2\theta \int_{\sigma=0}^{\infty} d\sigma\, \sigma^2 \int_{\rho=-\infty}^{\infty} \mathrm{sinc}(\rho T)\, \exp(i\rho r/c) d$$

$$\times \hat{v}_T\left(\frac{\sigma+\rho/2}{c}\frac{x}{r}, \frac{\sigma+\rho/2}{c}\frac{y}{r}; 0; \sigma+\rho/2\right)$$

$$\times \hat{v}_T^*\left(\frac{\sigma-\rho/2}{c}\frac{x}{r}, \frac{\sigma-\rho/2}{c}\frac{y}{r}; 0; \sigma-\rho/2\right), \quad (13)$$

where the dependence of the amplitude on the difference-frequency coordinate, ρ, has been dropped, since for quasi-monochromatic radiation $\sigma^2 \gg |\rho^2/4|$.

Now, using the defining transform relation of Eq. (8), we write the spatial-correlation function at the source ($z = 0$) where

$$\hat{v}_T\left(\frac{\sigma+\rho/2}{c}\frac{x}{r}, \frac{\sigma+\rho/2}{c}\frac{y}{r}; 0; \sigma+\rho/2\right)$$

$$\times \hat{v}_T^*\left(\frac{\sigma-\rho/2}{c}\frac{x}{r}, \frac{\sigma-\rho/2}{c}\frac{y}{r}; 0; \sigma-\rho/2\right)$$

$$= \frac{1}{(2\pi)^2} \iiiint_{-\infty}^{\infty} v_T(\xi_1,\eta_1;0;\ \sigma+\rho/2)$$

$$\times v_T^*(\xi_2,\eta_2;0;\ \sigma-\rho/2)$$

$$\times \exp\left[-i\left(\frac{\sigma+\rho/2}{c}\right)\left(\xi_1 \frac{x}{r} + \eta_1 \frac{y}{r}\right)\right]$$

$$\times \exp\left[i\left(\frac{\sigma-\rho/2}{c}\right)\left(\xi_2 \frac{x}{r} + \eta_2 \frac{y}{r}\right)\right] d\xi_1 d\xi_2 d\eta_1 d\eta_2 \ . \qquad (14)$$

Now the intermediate-average spatial-correlation function,

$$v_T(\xi_1,\eta_1;0;\ \sigma+\rho/2)\ v_T^*(\xi_2,\eta_2;0;\ \sigma-\rho/2)\ ,$$

when considered with the filtering action of the sinc function of Eq. (13), will have an effective contribution only for the low-frequency components formed by the difference-frequency terms $\sim 1/(2T)$ Hz or less.

In addition, we assume the mode population to be a slowly varying function of σ, since $\sigma \gg \rho/2$, and thus we write

$$[v_T(\xi_1,\eta_1,0;\ \sigma+\rho/2)\ v_T^*(\xi_2,\eta_2,0;\ \sigma-\rho/2)]_{\text{low freq.}}$$

$$= A(\xi_1,\eta_1;\ \sigma+\rho/2)\ A(\xi_2,\eta_2;\ \sigma-\rho/2)$$

$$\exp[i\phi\ (\xi_1-\xi_2,\ \eta_1-\eta_2;\rho) \qquad (15a)$$

$$= A(\underline{\xi}_1,\sigma)\ A(\underline{\xi}_2,\sigma)\ H(\rho)\ \exp[i\phi\ (\underline{\xi}_1-\underline{\xi}_2;\rho)]\ , \qquad (15b)$$

where

$$H(\rho) = \begin{cases} 1 & \text{for } A(\underline{\xi};\sigma\pm\rho/2) \neq 0 \\ 0 & \text{otherwise.} \end{cases}$$

We note that, in Eq. (15b), a spatial-vector symbolism has been adopted to shorten the notation, where $\underline{\xi} \equiv \xi\hat{i} + \eta\hat{j}$. Equation (15) indicates the loss of the optical-frequency phase, whereas the phase of the intensity envelope formed by temporal beat modes is preserved. The degree to which the phase of this envelope is detected depends on the bandwidth of the source and the detector resolution, 2T.

Essentially, the approximation of Eq. (15b) was made by Hanbury Brown and Twiss, except for the defining of the H function. Its introduction is brought about by the description of narrow-band sources by terms in $A(\underline{\xi},\sigma)$. For a thermal source of relatively large bandwidth, many of the higher beat-frequency (ρ) components will not be resolved by the system response ($\sim$2T) described by the sinc function. In addition, the beat spectrum will be a continuous function. For a laser source exhibiting a series of axial modes, however, the complete difference-frequency domain might lie entirely within the system response but be piece-wise continuous in its extent.

Relative to the representation of the intermediate average by the form of Eq. (15), we wish to reiterate a statement made following Eq. (1) that the intermediate-averaging process may bear little resemblance to the infinite time average, even so far as the detail of the ampl[illegible] terms, $A(\underline{\xi},\sigma)$. This situation would be serious if our intent were to infer, for example, the time-frequency statistics of the source. But in the present concept, we desire only to infer the spatial properties of the source. If we consider a multi-axial-mode laser beam scattered from a spatially rough surface, the lack of correspondence between the two averages is unimportant, for all such mode history is integrated out; all areas of the scatterer see the same mode characteristics. Any mode fluctuation would be seen simply as a variation in total received power from one sample to the next. Here, we simply require for one detector-resolution time over a spatial domain that the process of Eq. (15) exhibit a minimum of two temporal modes [to maintain the phase term $\phi(\underline{\xi}_1-\underline{\xi}_2,\rho)$ with sufficient mode population (reflected in the

amplitude terms $A(\underline{\xi},\sigma)$}] such that quantum noise in both the carrier wave and the detector can be ignored.

Using the results of Eq. (15) in Eq. (13) and taking $\theta \ll 1$, we write

$$I(\underline{x},T) = \left(\frac{1}{cr}\right)^2 \int_0^\infty \sigma^2 d\sigma \int_{-\infty}^{\infty} d\rho \exp(i\rho r/c) \operatorname{sinc}(\rho T) H(\rho)$$

$$\times \iiiint_{-\infty}^{\infty} A(\underline{\xi}_1,\sigma) A(\underline{\xi}_2,\sigma) \exp[i\phi(\underline{\xi}_1-\underline{\xi}_2;\rho)]$$

$$\exp\left[-i\left(\frac{\sigma+\rho/2}{rc}\right)(\underline{\xi}_1\cdot\underline{x})\right]$$

$$\exp\left[i\left(\frac{\sigma-\rho/2}{rc}\right)(\underline{\xi}_2\cdot\underline{x})\right] d\underline{\xi}_1 d\underline{\xi}_2 \ . \tag{16}$$

We now have the self-intensity in the far field as a double integral over sum- and difference-frequency components as well as two, two-dimensional spatial Fourier transforms over the source.

Following Goodman,[14] we argue that the received field at any point in the far zone consists of a sum of random-amplitude, random-phase, complex phasors contributed by the elementary scatterers. If the size of the scattering area is large enough to include many point scatterers (or there are enough elementary coherence areas composing the source), the central limit theorem may be used to conclude that the electric field in the detection plane is a gaussian random process in a spatial sense.

Using the form of Eq. (16) and its property of spatial gaussian statistics, we are ready to form the fourth-order correlation function in the far zone.

IV. FOURTH-ORDER FIELD CORRELATION IN THE FAR ZONE

Basic to the theory of intensity correlation is the ability to represent higher-order field correlations (fourth, sixth, etc.) in terms of the first and second. This is due to a well-known property of gaussian processes.[15] However, in the work of Hanbury Brown and Twiss, this property was never explicitly used, although it was discussed later by Wolf[5] as a plausibility argument for their work.

To form the fourth-order correlation function, we can proceed by writing the two-point product of intensities in the far field using Eq. (16) in a manner similar to that of Hanbury Brown and Twiss. However, to develop an approach adaptable to arbitrary orders of correlations, as well as to allow consideration of scattering surfaces with arbitrary roughness, we start by examining a relationship valid for real fourth-order gaussian processes[5,15] (here in the real electric-field spatial variable), where

$$2\,\langle V(\underline{x}_1)\; V\,(\underline{x}_2)\rangle^{2} = \langle \Delta V^2(\underline{x}_1)\; \Delta V^2(\underline{x}_2)\rangle \tag{17a}$$

$$= \langle \Delta I(\underline{x}_1)\; \Delta I(\underline{x}_2)\rangle \tag{17b}$$

and where we define

$$\Delta V^2(\underline{x}_n) \equiv V^2(\underline{x}_n) - \langle V^2(\underline{x}_n)\rangle \tag{18a}$$

and

$$\Delta I(\underline{x}_n) \equiv I(\underline{x}_n) - \langle I(\underline{x}_n)\rangle\,. \tag{18b}$$

Equation (17b) follows from the definition of intensity, and the angle brackets ⟨⟩ indicate an ensemble (spatial) average, not the more usual time average.

Using the identity

$$\langle \Delta I(\underline{x}_1)\ \Delta I(\underline{x}_2)\rangle = \langle I(\underline{x}_1)\ I(\underline{x}_2)\rangle - \langle I(\underline{x}_1)\rangle\ \langle I(\underline{x}_2)\rangle \ , \tag{19}$$

we can write

$$\langle I(\underline{x}_1)\ I(\underline{x}_2)\rangle = 2\langle V(\underline{x}_1)\ V\ (\underline{x}_2)\rangle^2 + \langle I(\underline{x}_1)\rangle\ \langle I(\underline{x}_2)\rangle \ . \tag{20}$$

We can conclude from Eq. (20) that the second-order intensity correlation is composed of two terms, of which one forms the square of the second-order field correlation; the other is a spatial dc term, of no value here. Hanbury Brown and Twiss eliminated a similar temporal term by means of a dc block in their electronic apparatus. The mathematical origin of this term can be observed in Eq. (16) for the case $\rho = 0$, corresponding to the infinite time average. For $\rho = 0$, the product of intensities in the far field for just two points is a constant; but, as the mean position of the points is translated spatially, the product varies and thus for the spatial averaging case, terms in $\rho = 0$ must be evaluated.

Similarly, for the complex field variable we can write, using Eqs. (6) and (16),

$$\langle I(\underline{x}_1,T)\ I(\underline{x}_2,T)\rangle = \left|\langle \Gamma(\underline{x}_1,\underline{x}_2;\ 0;T)\rangle\right|^2 \tag{21a}$$

$$= \frac{1}{(\pi r)^4}\left| \int_0^{\infty} \sigma^2 d\sigma \int_{-\infty}^{\infty} d\rho\ \mathrm{sinc}(\rho T)H(\rho)\ \exp(i\rho r/c)\right.$$

$$\times \iiiint_{-\infty}^{\infty} \langle A(\underline{\xi}_1,\sigma)\ A(\underline{\xi}_2,\sigma)\ \exp[i\phi(\underline{\xi}_1-\underline{\xi}_2;\rho)]\rangle$$

$$\times \exp\left[-i\left(\frac{\sigma+\rho/2}{rc}\right)(\underline{\xi}_1\cdot\underline{x}_1)\right]$$

$$\times \exp\left[i\left(\frac{\sigma-\rho/2}{rc}\right)(\underline{\xi}_2\cdot\underline{x}_2)\right] d\underline{\xi}_1\, d\underline{\xi}_2\Bigg|^2 , \qquad (21b)$$

where we have interchanged the order of integration and averaging.

Next, we make the following transformation to spatial center-of-mass coordinates where

$$\underline{f} = \underline{\xi}_1 - \underline{\xi}_2 \qquad \text{and} \qquad \underline{g} = \frac{1}{2}\,(\underline{\xi}_1 + \underline{\xi}_2) \ . \qquad (22)$$

Introducing these into Eq. (21b), following some algebra, we find (suppressing constant terms)

$$\langle I(\underline{x}_1,T)\ I(\underline{x}_2,T)\rangle = \frac{1}{(cr)^4}\left|\int_0^\infty \sigma^2 d\sigma \int_{-\infty}^{\infty} \mathrm{sinc}(\rho T) W(\rho)\, d\rho\, \exp(i\rho r/c)\right.$$

$$\times \iiiint_{-\infty}^{\infty} \langle A(\underline{g}+\underline{f}/2,\sigma)\ A(\underline{g}-\underline{f}/2,\sigma)\ \exp[i\phi(\underline{f},\rho)]\rangle$$

$$\times \exp\left[-i\left(\frac{\sigma+\rho/2}{rc}\right)[(\underline{g}+\underline{f}/2)\cdot\underline{x}_1]\right]$$

$$\left.\times \exp\left[i\left(\frac{\sigma-\rho/2}{rc}\right)[(\underline{g}-\underline{f}/2)\cdot\underline{x}_2]\right] d\underline{f}\, d\underline{g}\right|^2 \qquad (23a)$$

$$\simeq \frac{1}{(cr)^4} \left| \int_0^\infty \omega^2 d\omega \int_{-\infty}^\infty \mathrm{sinc}(\rho T) H(\rho) d\rho \right.$$

$$\times \iint_{-\infty}^{\infty} I(\underline{\xi},\omega) \exp\left[-i\frac{k}{r}\,(\underline{x}_1-\underline{x}_2)\cdot\xi\right] d\xi$$

$$\left. \times \iint_{-\infty}^{\infty} C(\underline{f}) \exp\left[-i\frac{k}{2r}\,(\underline{x}_1+\underline{x}_2)\cdot\underline{f}\right] d\underline{f}\right|^2 \tag{23b}$$

$$\propto \left| \hat{I}\left(\frac{k}{r}(\underline{x}_1-\underline{x}_2)\right)\right|^2 \left| \hat{C}\left(\frac{k}{2r}(\underline{x}_1+\underline{x}_2)\right)\right|^2 . \tag{23c}$$

In going from Eq. (23a) to Eq. 23b), we have dropped the Fourier-transform terms in the difference frequency variable ρ, since they are clearly negligible in the far field. The ensemble average has been expressed as a product of two terms: $C(\underline{f})$ is a normalized correlation function[14] describing the coherence interval over a rough surface, and $I(\underline{\xi},\omega)$ is the intensity in the global-spatial variable. We have made the reasonable assumption that the field amplitude is constant within a given coherence area of the source; specifically, for $\underline{f}$ sufficiently small that $C(\underline{f}) \neq 0$, $A(\underline{g}+\underline{f}/2,\sigma) \simeq A(\underline{g}-\underline{f}/2,\sigma) \simeq A(\underline{\xi},\sigma)$. This final approximation is made under the assumption that the coherence area of the source is small relative to the total source area, an assertion already made in an earlier argument for gaussian statistics. In Eq. (23c), the hat ^ indicates a two-dimensional spatial-Fourier transform.

In order to describe a spatially incoherent surface, $C(\underline{f})$ is usually allowed to assume the role of a δ function.[16] Thus, Eqs. (23) are reduced to a single integral in two space. Given this form, we see from Eqs. (23) that the ensemble-averaged, two-pont intensity correlation in the far field is proportional to the modulus of the spatial Fourier transform across a spatially rough surface, assuming a sufficiently short exposure time 2T. However, only for the case that the intensity distribution over the source has even symmetry can the phase of the spatial transform be inferred and used to invert uniquely Eqs.(23) to derive the intensity distribution on the source, $I(\underline{\xi},\omega)$.

Finally, the form of Eq. (23c) shows explicitly that the two-point ensemble-average intensity correlation in the far field is proportional to the product of two spatial power spectra. The spatial power spectrum

$$\left| \hat{I}\left(\frac{k}{r}(\underline{x}_1-\underline{x}_2)\right) \right|$$

is multiplied by

$$\left| \hat{C}\left(\frac{k}{2r}(\underline{x}_1+\underline{x}_2)\right) \right|^2 ,$$

the spatial power spectrum of the correlation function describing the surface roughness. If the surface is sufficiently rough that this function approximates a δ function, then the transform is essentially constant, and all spatial frequencies of the source can be inferred. However, as the correlation interval increases,

$$\left| \hat{C}\left(\frac{k}{2r}(\underline{x}_1+\underline{x}_2)\right) \right|^2$$

acts to band limit the detectable spatial spectrum of the source. This effect is discussed, for example, by Kinsly[17] for the case of microdensitometer imaging with partially coherent light.

V. AN EXPERIMENT

To illustrate the above ideas, we suggest a simple experiment embodying these mathematical ideas. A helium-neon laser in single-axial-mode configuration is used to transilluminate a symmetrical source (Fig. 1). The source has a random-phase character to obtain spatial incoherence. The far-zone intensity pattern is recorded by film, using an exposure time less than the reciprocal of the source bandwidth.[18]

Next, the film is developed so that it is linear in intensity and used to make two identical positive transparencies. The positives are then placed in a collimated beam (Fig. 2) to form the correlated intensity over an averaging area. The signal transmitted by the transparency pair is optically Fourier transformed, a dc block is inserted to remove the unwanted average term, and the total remaining irradiance is measured. This signal represents the mathematical expression given by Eqs. (23) for the transparency spatial lag, $\underline{d} = \underline{x}_1 - \underline{x}_2$. Since the source is known, *a priori*, to be symmetrical, the transform of the source intensity is real. The square root of the correlation signal is proportional to the spectrum, which is then known as a function of spatial lag. Finally, this two-dimensional signal is Fourier transformed by machine to give the scaled source irradiance.

VI. SUMMARY AND CONCLUSIONS

Developing an intermediate-average, mutual coherence function as a starting point, we have derived an expression for the two-point intensity correlation in the far field, independent of time averaging except for the temporal resolution of the detector. This result is valid for narrow-band, high-intensity light scattered from a spatially rough surface of arbitrary coherence area.

There are a number of special benefits from detecting images by the technique of intensity correlation. (i) The method is relatively insensitive to the effects of atmospheric scintillation.[2] (ii) Because

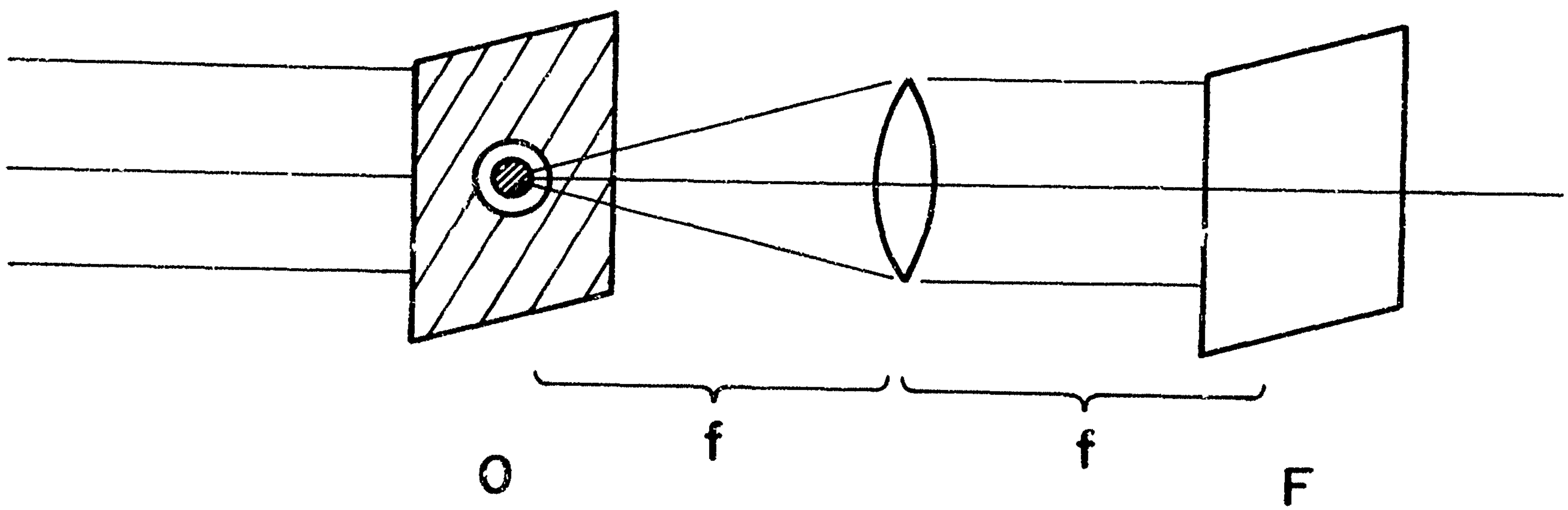

Fig. 1. A symmetrical object (O) is illuminated with collimated light from a single-axial-mode laser. Ground glass is introduced at plane O to achieve spatial incoherence. The far-field intensity pattern is recorded by film at plane F. The lens focal length indicated by f.

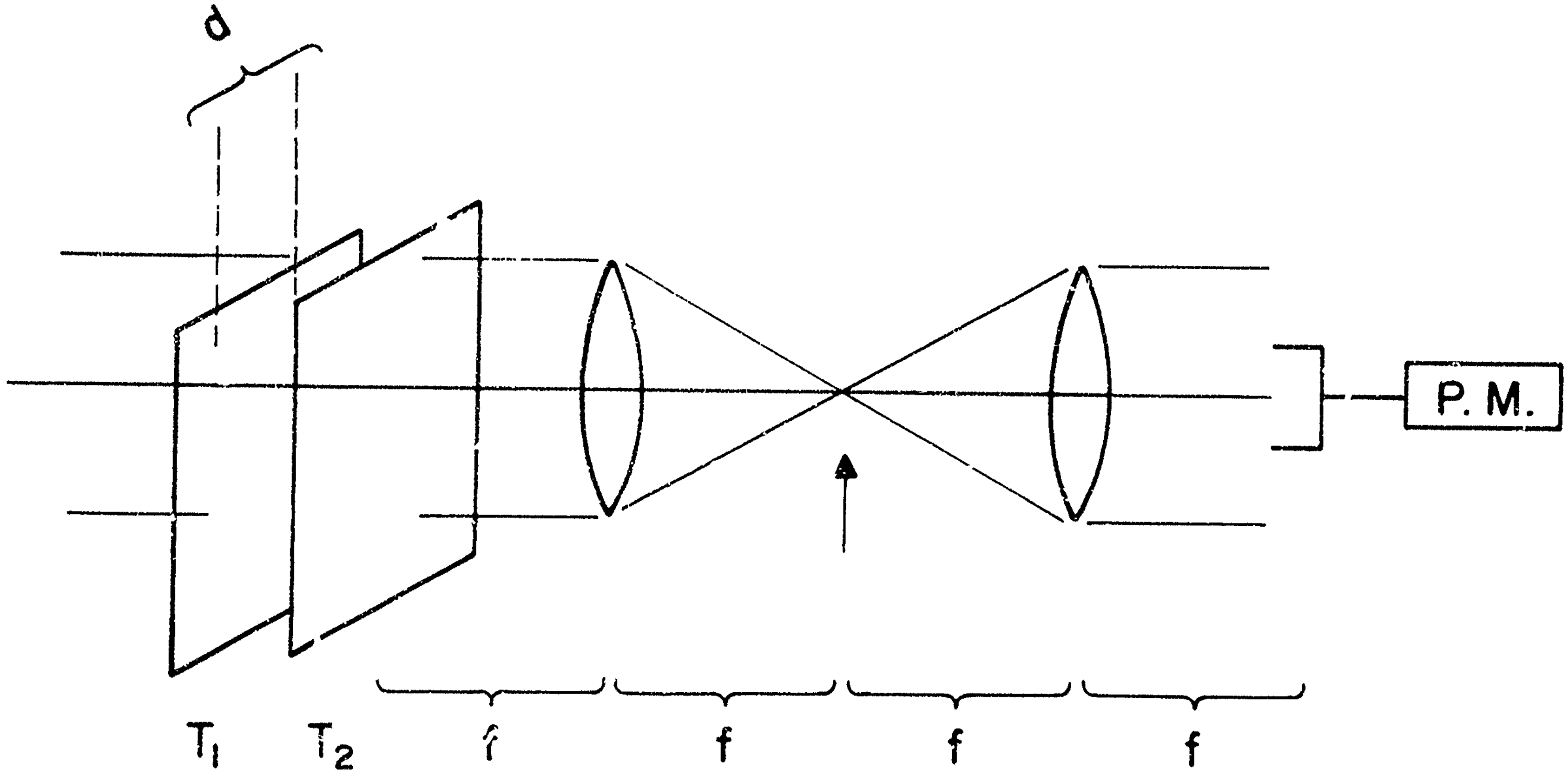

Fig. 2. A pair of transparencies, T_1 and T_2, are made from film record in plane F of Fig. 1. Intensity autocorrelation function is measured by power meter for spatial lag $d=\underline{x}_1-\underline{x}_2$; dc block is inserted between lenses at arrow to remove unwanted signal.

the signal is detected in the spatial-transform domain, high-frequency detail about the scattering surface translates to large spatial lags in the far field. This result could be particularly important at frequencies where detector resolution is not well developed. (iii) A special advantage to intensity interferometry in the spatial domain is the utilization of gaussian statistics in the spatial (not temporal) sense. By this method, sources with nongaussian time statistics (such as single-axial-mode lasers) can be utilized. (iv) Still another advantage of spatial detection is that images of moving surfaces can be formed using brief exposures.

We have therefore shown that, given a symmetrical, spatially incoherent source illuminated by high-intensity light, the far-zone intensity pattern can be used to form the optical image of the source if the signal is recorded with sufficiently short time resolution.

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7. Ref. 3b, Eq. (2.1).

8. What is sufficient obviously depends on the bandwidth of the source. For a thermal source, most beat frequencies are too fast to be resolved even with the megacycle response of the Hanbury Brown-Twiss apparatus. For a single-mode laser, all beat frequencies could be less than 100 Hz.

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16. We emphasize that here spatial coherence is defined in terms of an ensemble (spatial) rather than the more usual time-average sense.

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18. A helium-neon laser in single-axial-mode operation has an optimal bandwidth of about 100 Hz. An exposure of one msec or less would resolve the phase of all the beat frequencies.